TMDs in Lattice QCD

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supported by

excellence cluster universe

PhH, B. Musch, J. Negele, A. Schäfer, arXiv:0908.1283
B. Musch, PhD thesis arXiv:0907.2381

Why study TMDs on the lattice? Motivation (I)

lattice QCD is a systematic „ab initio“-approach to non-perturbative physics

cross-sections (SIDIS, DY-production), asymmetries, T-odd effects

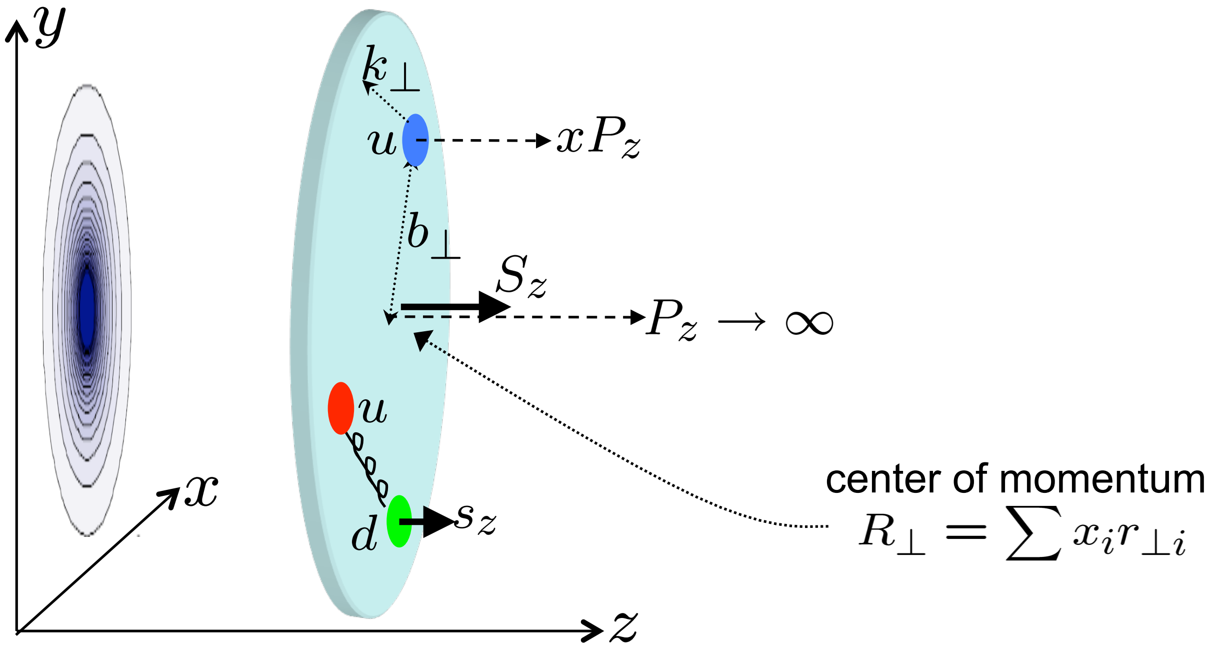
QCD factorization, (separation of) scales, regularization&renormalization, gauge invariance, gauge link structures

there is no unique, commonly accepted exact definition of TMDs in terms of (GI) quark- and gluon operators available

Mulders et al.(„LO“); Anelmino et al., Radicci, Bachetta et al., … („LO“)
Collins; Collins&Metz; Collins&Hauptmann; Ji,Ma&Yuan („NLO“)
Cherednikov&Stefanis („NLO“)
Chay (EFT)

lattice QCD may help to explore different ansätze and definitions of TMDs

Motivation (II): Hadron structure



generalized parton distributions (GPDs) — $H(x, b_\perp) \overset{\text{FT}}{\leftrightarrow} H(x, \xi = 0, t)$ — $b_\perp \leftrightarrow -\Delta_\perp \widehat{=} |t|^{\frac{1}{2}}$

Burkardt PRD 2000

~~Fourier-transformation~~

transverse momentum dependent PDFs — $f(x, k_\perp) \overset{\text{FT}}{\leftrightarrow} A(l \cdot P, l^2)$ — $k_\perp \leftrightarrow l_\perp$, $x \leftrightarrow l \cdot P$

$\langle P | \overline{q}(l) \Gamma U q(0) | P \rangle$

Lattice QCD calculations of hadron structure

systematic „ab initio"-approach, but

- statistical errors from MC integration
- discretization and finite volume errors/effects
- contaminations from excited states
- large quark masses $m_\pi (\propto \sqrt{m_q}) \gtrsim 300$ MeV
- large minimal non-zero momenta $p_{\text{min}} = \frac{2\pi}{aL} \approx 300$ MeV

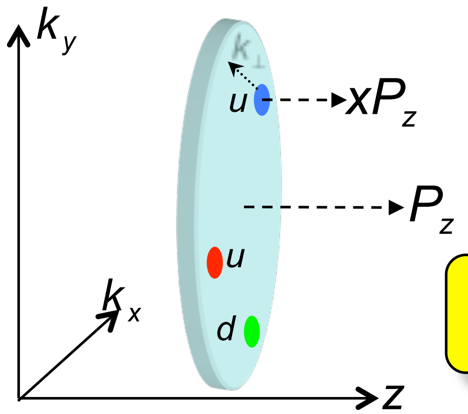
models of TMDs are interesting and important, but

Lattice QCD is different from model calculations

→ approximations can be continuously improved

mainly limited by computational resources

Transverse momentum dependent PDFs - formalism



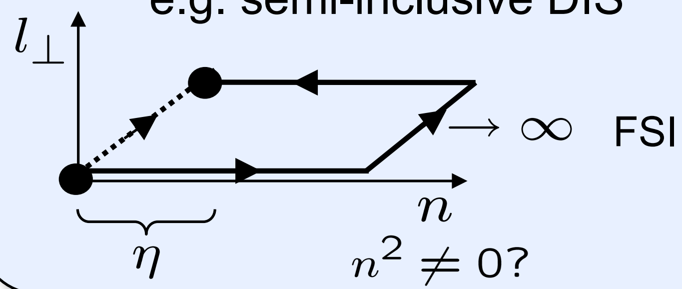
$$f(x,k_\perp) \propto \int d\eta\, d^2 l_\perp\, e^{ix\eta P} e^{-i l_\perp \cdot k_\perp} \langle P|\bar{q}(-\tfrac{\eta}{2}n, l_\perp)\Gamma U q(\tfrac{\eta}{2}n, 0_\perp)|P\rangle$$

$$f(x,k_\perp) \propto \int d(l\cdot P)\, d(l^2)\, e^{ix(l\cdot P)} J_0(\sqrt{-l^2}|k_\perp|) A_2(l\cdot P, l^2)$$

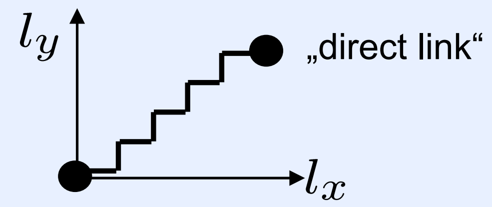
complex amplitude A_2

form/choice of Wilson line U(l)

e.g. semi-inclusive DIS



exploratory lattice study

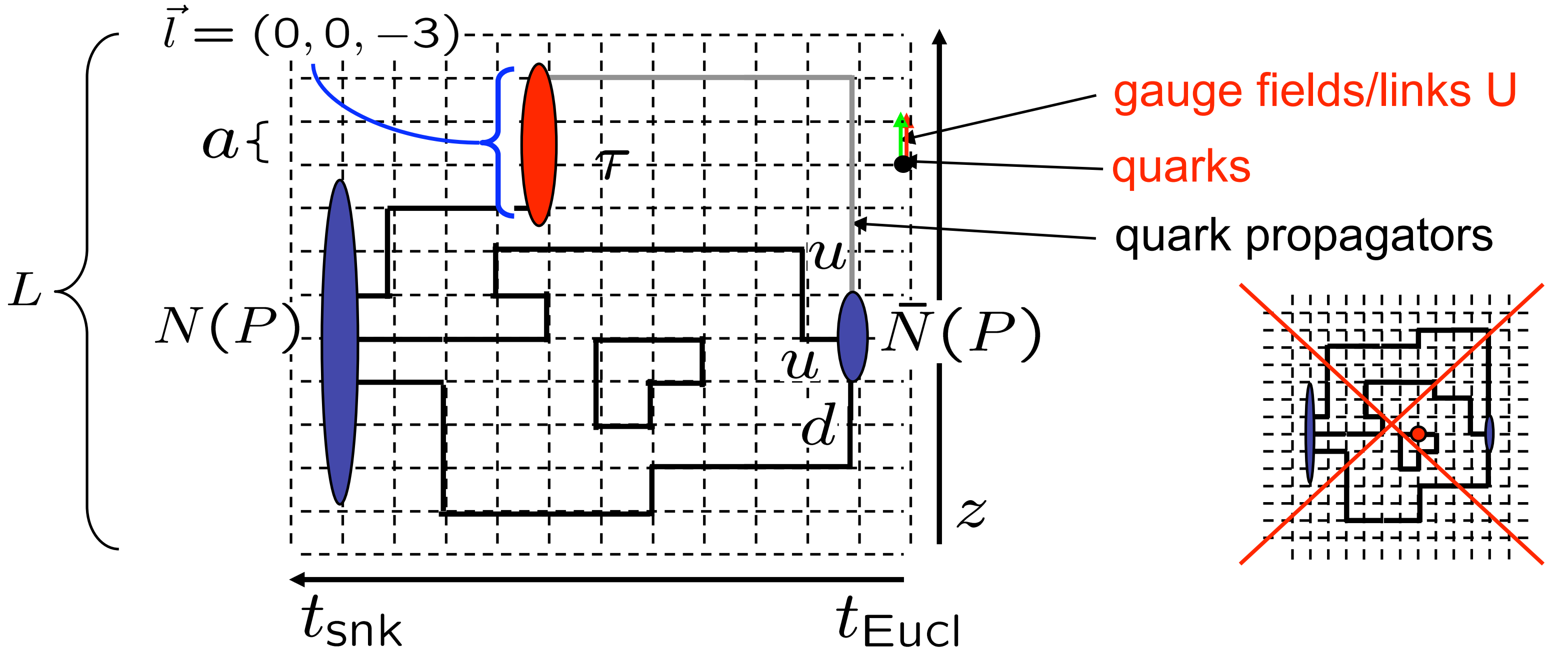


precise form?

many different proposals on the market

„process-independent“

Lattice QCD calculations of hadron structure



= vector-, axialvector-, quark spin flip-operator

$$C_{3pt}(P, t_{\text{snk}}, \tau; l) \leftrightarrow Z e^{-E t_{\text{snk}}} \langle P, \Lambda' | \underbrace{\mathcal{O}(l)} | P, \Lambda \rangle \propto \tilde{A}_i(l^2, l \cdot P)$$

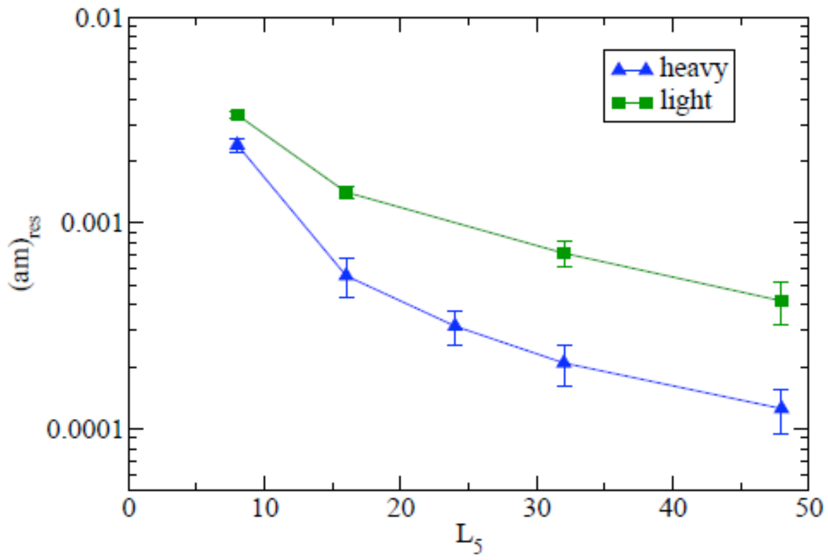
$$\langle q_2 \bar{q}_1 \rangle \propto \int DADqd\bar{q} e^{iS[q,\bar{q},A]} \rightarrow \left[\int DU e^{-S[U]} \det D[U]\right] D^{-1}_{1\rightarrow 2}[U] \approx \frac{1}{N} \sum_{i=1}^{N} D^{-1}_{1\rightarrow 2}[U_i]$$

compute the path-integral numerically

Phys. Rev. D77, 094502

Lattice parameters – LHPC/MILC

- domain-wall-fermions on a staggered "Asqtad" staggered sea ("hybrid" formalism) with HYP-smearing
- use of staggered quarks is "a matter of taste"
- $N_f = 2+1$, but only connected contributions
- $Ls = 16$, $m_{res} \leq 0.1 m_q$
- inverse lattice-spacing is $a^{-1} \approx 1.6$ GeV
- pion masses as low as 300MeV in volumes $\leq (3.5\text{fm})^3$
- one projector $\widetilde{\Gamma}_{pol} = \frac{1}{4}(1+\gamma_0)(1-\gamma_5\gamma_3)$
- two sink-momenta $p' = (0,0,0),(-1,0,0)$

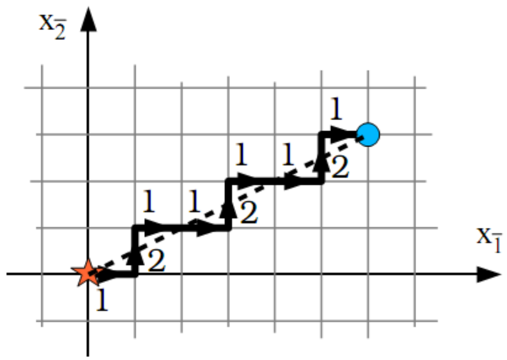


so far only one ensemble analyzed

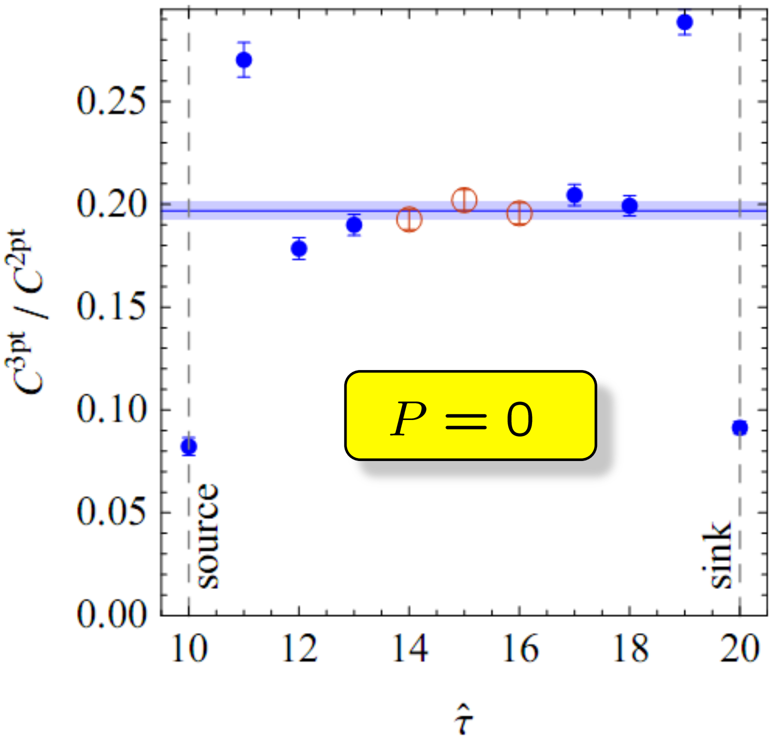
dataset	Ω	#	$(am)_q^{\text{Asqtad}}$	$(am)_q^{\text{DWF}}$	$(am)_\pi^{\text{Asqtad}}$	$(am)_\pi^{\text{DWF}}$	$(am)_N^{\text{Asqtad}}$	$(am)_N^{\text{DWF}}$	m_π^{DWF} [MeV]
1	$20^3 \times 32$	425	0.050/0.050	0.0810	0.4836(2)	0.4773(9)	1.057(5)	0.986(5)	758.9(1.4)
2		350	0.040/0.050	0.0478	0.4340(3)	0.4293(10)	1.003(3)	0.938(8)	682.6(1.6)
3		564	0.030/0.050	0.0644	0.3774(2)	0.3747(10)	0.930(3)	0.869(6)	595.8(1.6)
4		486	0.020/0.050	0.0313	0.3109(2)	0.3121(11)	0.854(3)	0.814(7)	496.2(1.7)
5		655	0.010/0.050	0.0138	0.2242(2)	0.2243(10)	0.779(6)	0.730(12)	356.6(1.6)
6	$28^3 \times 32$	270	0.010/0.050	0.0138		0.2220(9)		0.766(15)	352.3(1.4)
7	20^3x32	460	0.007/0.05			0.1842(7)			292

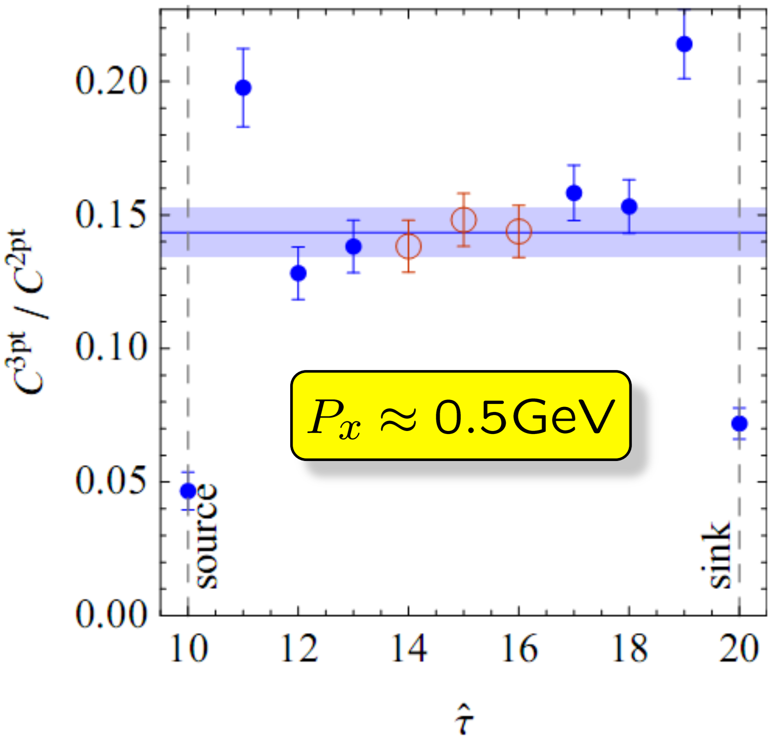
Plateaus

$$\frac{C_{3pt}(P,\tau,t_{\text{snk}};l)}{C_{2pt}(P,t_{\text{snk}})} = f(l,P;\tau,t_{\text{snk}})$$



$l \approx 0.8\text{fm}$





Transverse momentum dependent PDFs - formalism

$$\langle P|\bar{q}(-\frac{\eta}{2}n, l_\perp)\Gamma U q(\frac{\eta}{2}n, 0_\perp)|P\rangle$$

$$\widetilde{A}_2(l^2, l\cdot P), \widetilde{A}_3, \widetilde{A}_6, \widetilde{A}_7, \widetilde{A}_8, \widetilde{A}_{9m}, \widetilde{A}_{10}, \widetilde{A}_{11}$$

Mulders, Tangermann NPB 1996
Boer, Mulders PRD 57 (1998)

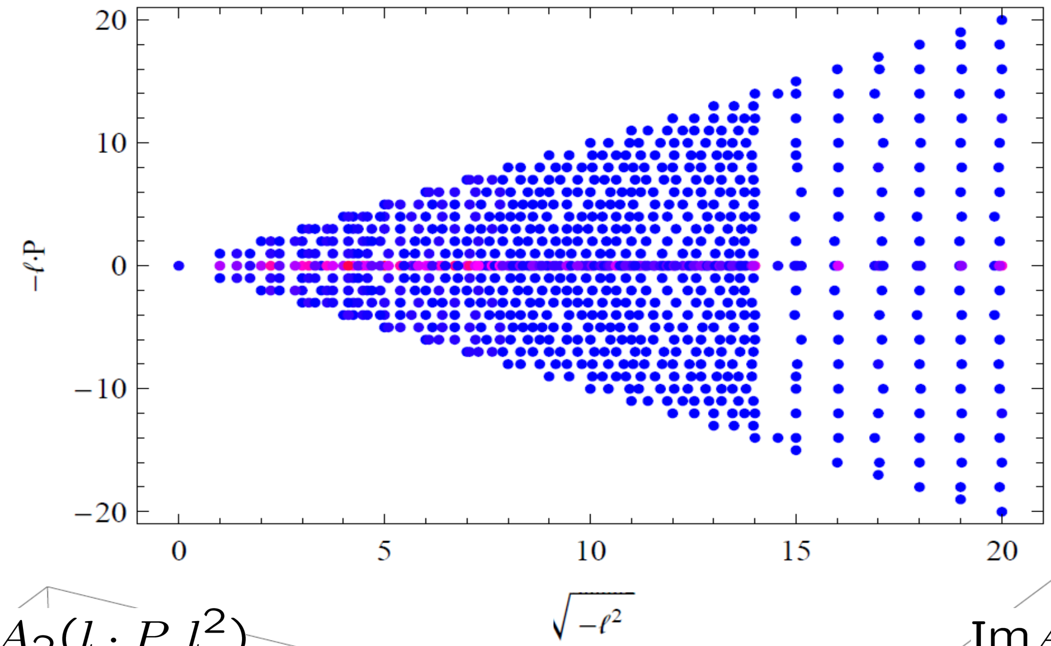
Fourier-transformation

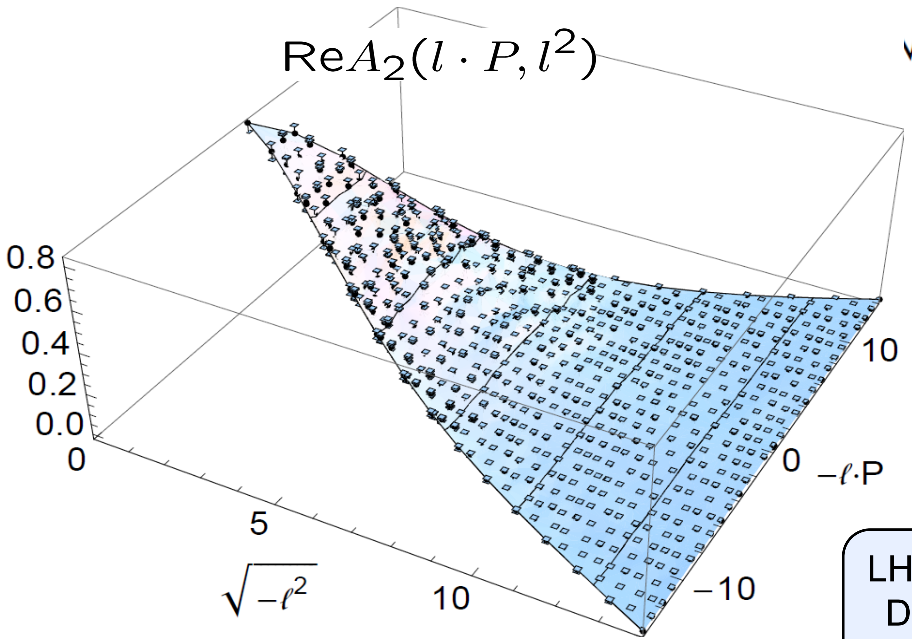
$$l^2 \leftrightarrow k_\perp^2,\; l\cdot P \leftrightarrow x$$

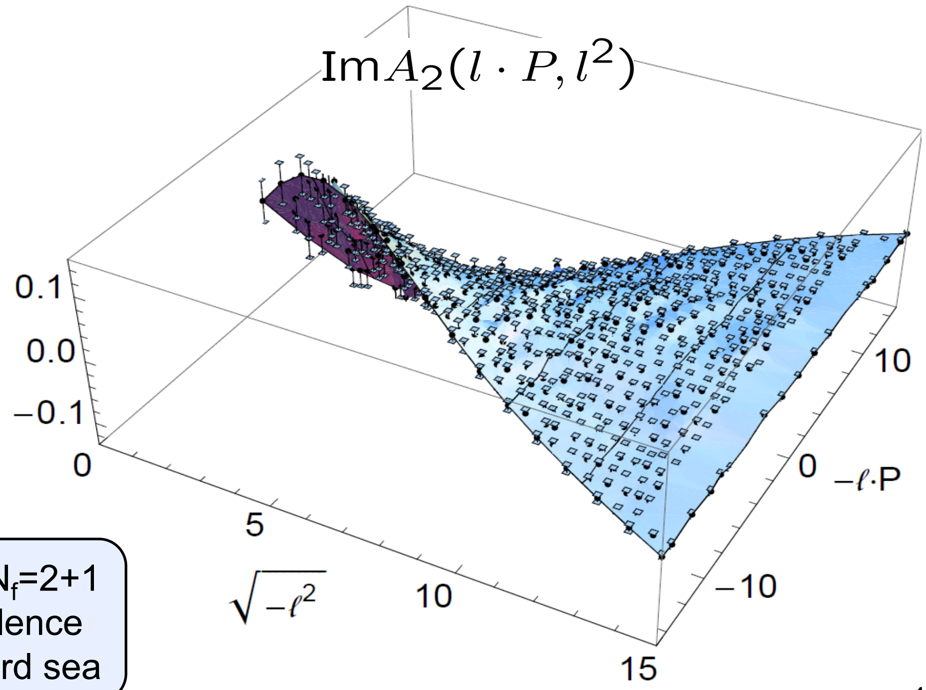
$$l\cdot P = 0 \leftrightarrow \int_{-1}^{1} dx$$

$$\begin{aligned} \widetilde{A}_2 &\leftrightarrow f_1(x, k_\perp^2), \\ \widetilde{A}_{6,7} &\leftrightarrow g_1,\; \widetilde{A}_7 \leftrightarrow g_{1T} \\ \widetilde{A}_{9m} &\leftrightarrow h_1,\; \widetilde{A}_{10,11} \leftrightarrow h_{1L}^\perp,\; \widetilde{A}_{11} \leftrightarrow h_{1T}^\perp \end{aligned}$$

Overview of selected results







LHPC, N_f=2+1
DW-valence
+staggerd sea

Renormalization

potential power-divergence

$$U[C_l] \propto e^{-\delta m\, l} = e^{-\frac{\widehat{\delta m}}{a}\, l}$$

$$V_{\bar{Q}Q}(R) = \lim_{T\to\infty} \partial_T \ln\langle W(R,T)\rangle = V^{\mathrm{ren}}_{\bar{Q}Q}(R) + 2\delta m$$

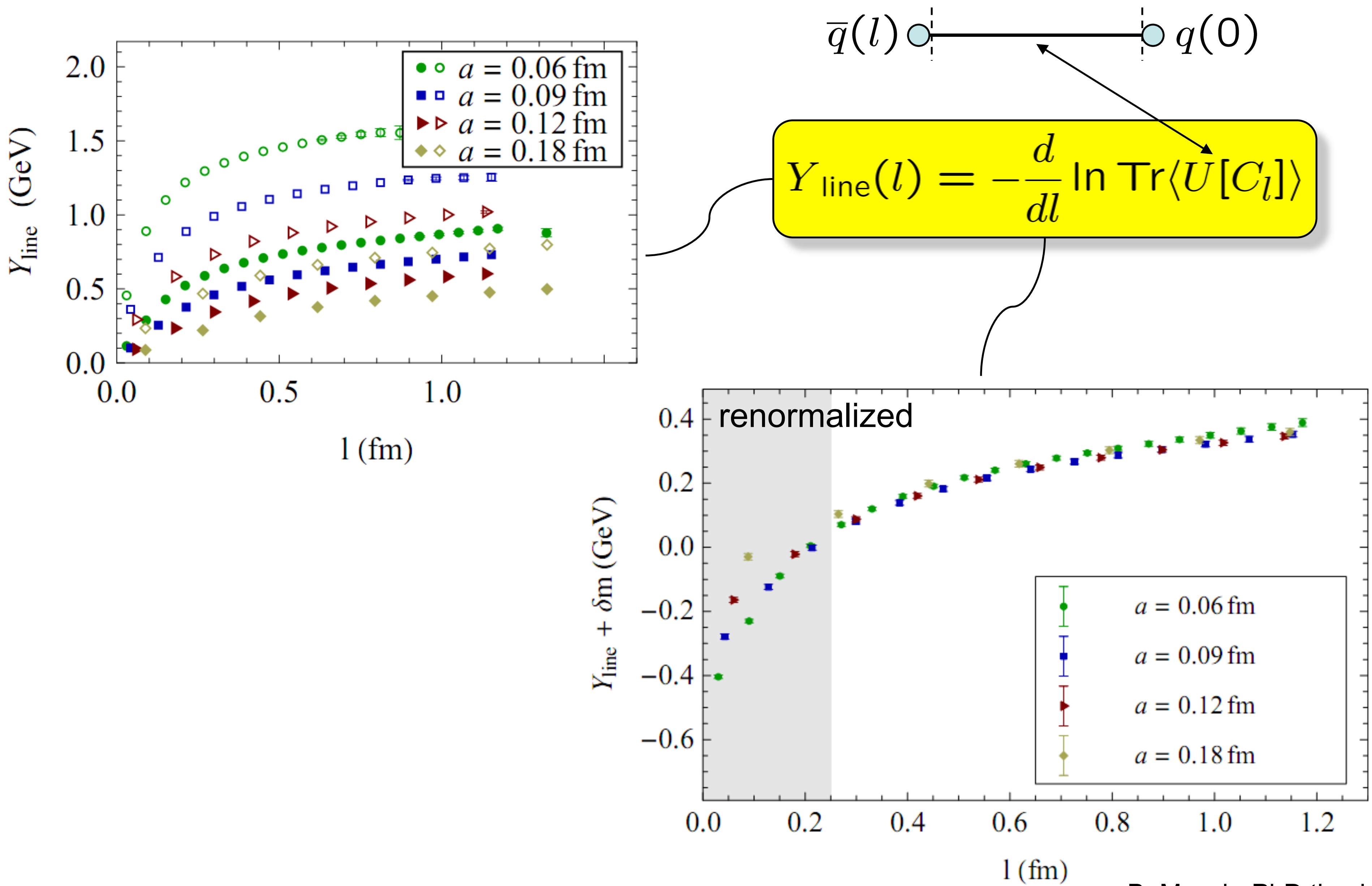
renormalization condition

$$V_{string}(R) = \sigma R - \frac{\pi}{12R} + C^{\mathrm{ren}}$$

$$C^{\mathrm{ren}} = 0$$

a[fm]	$\widehat{\delta m}$
0.12fm	0.1553(47)
0.08fm	0.1639(35)
0.06fm	0.1578(17)

Illustration of renormalization



Ph. Hägler, EINN TMD workshop, 2009

B. Musch, PhD thesis

„Regularization“ and multiplicative renormalization

Gaussian parametrization of the invariant amplitudes

$$2\tilde{A}_i(l^2, l \cdot P = 0) = c_i e^{l^2/\sigma_i^2}$$

restricted to l ~ 0.25 … 2 fm

reg. *potential* divergences at small l ⟺ large $k_\perp$

renormalize multiplicatively such that

$$\tilde{A}_2^{u-d,ren}(l^2 = 0, l \cdot P = 0) = F_1^{u-d}(t = 0) = 1$$

PhH, B. Musch et al.
arXiv:0908.1283

	c	$2/\sigma$ (GeV)
$\tilde{A}_2^u$	$2.0159(86) = f_{1,u}^{(0,0)}$	0.3741(72)
$\tilde{A}_2^d$	$1.0192(90) = f_{1,d}^{(0,0)}$	0.3839(78)
$\tilde{A}_6^u$	$-0.920(35) = -g_{1,u}^{(0,0)}$	0.311(11)
$\tilde{A}_6^d$	$0.291(19) = -g_{1,d}^{(0,0)}$	0.363(18)
$\tilde{A}_{9m}^u$	$0.931(29) = h_{1,u}^{(0,0)}$	0.3184(90)
$\tilde{A}_{9m}^d$	$-0.254(16) = h_{1,d}^{(0,0)}$	0.327(15)
$\tilde{A}_7^u$	$-0.1055(66) = -g_{1T,u}^{(0,1)}$	0.328(14)
$\tilde{A}_7^d$	$0.0235(38) = -g_{1T,d}^{(0,1)}$	0.346(36)
$\tilde{A}_{10}^u$	$-0.0931(73) = h_{1L,u}^{\perp(0,1)}$	0.340(14)
$\tilde{A}_{10}^d$	$0.0130(40) = h_{1L,d}^{\perp(0,1)}$	0.301(48)

$2/\sigma_i \hat{=} \langle k_\perp^2 \rangle^{1/2}$

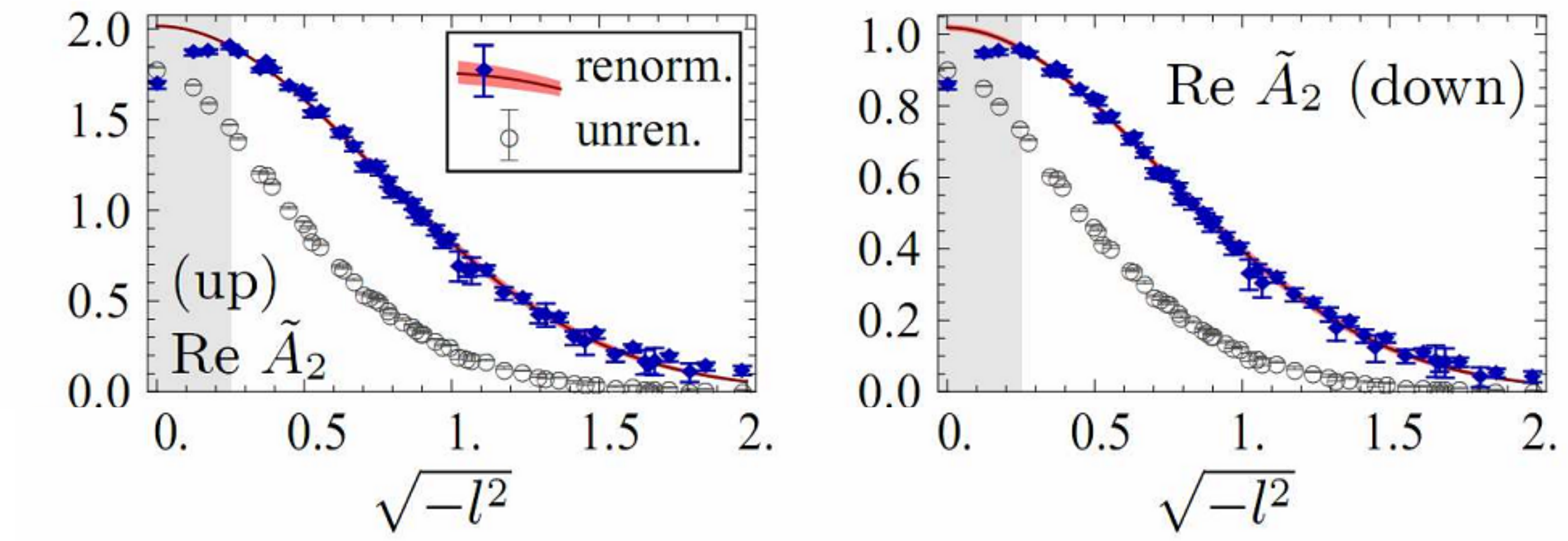
$\rightarrow g_A = 1.209(36)$

at this stage, better not compare quantitatively with TMD-phenomenology (e.g. Anselmino et al.)

l^2-dependence of invariant amplitudes (renormalized)

PhH, B. Musch et al.
arXiv:0908.1283

$\tilde{A}_2(l \cdot P = 0, l^2) \leftrightarrow f_1^{(n=1)}(k_\perp^2)$



l^2-dependence of invariant amplitudes (renormalized)

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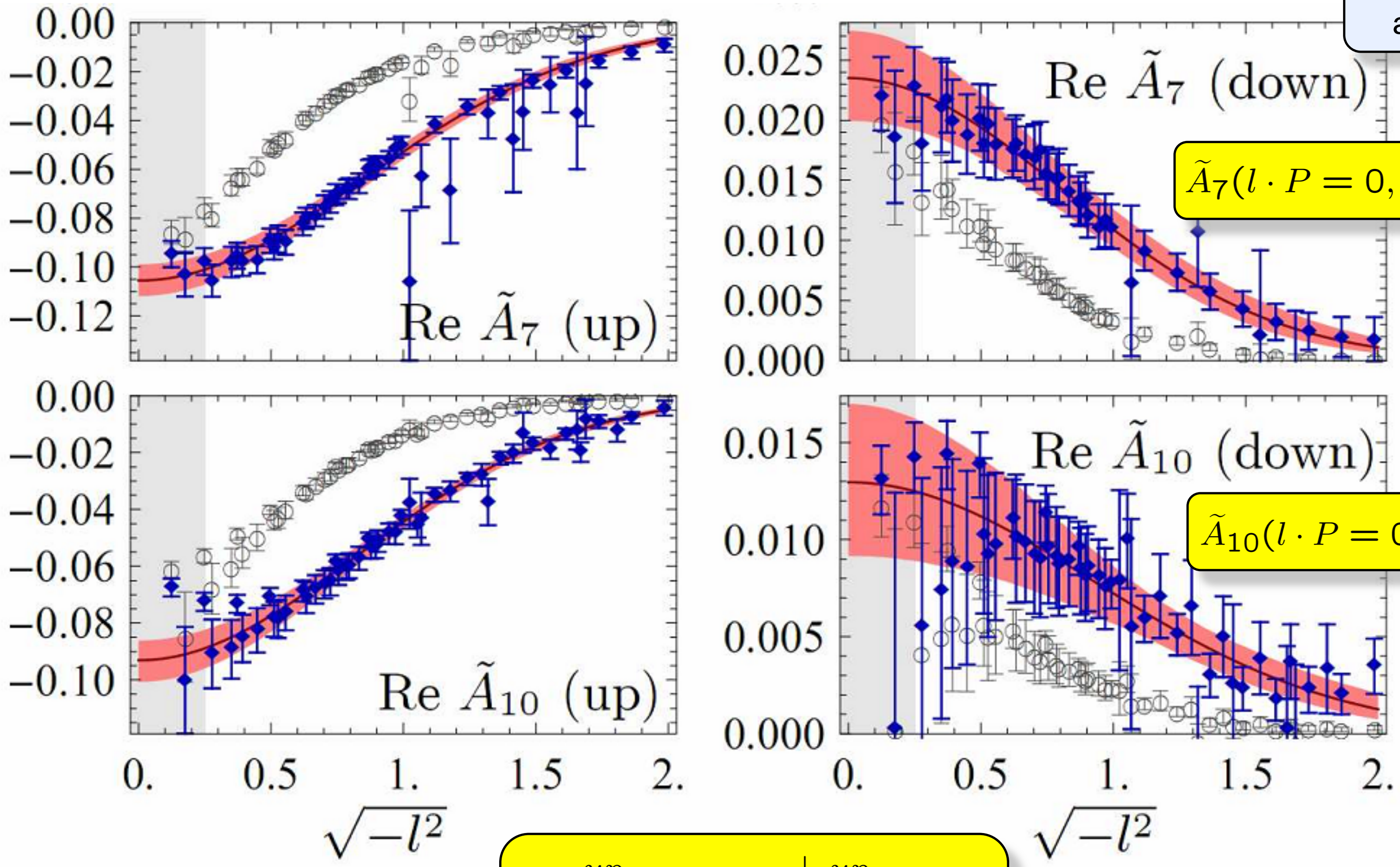


$$\tilde{A}_7(l \cdot P = 0, l^2) \leftrightarrow -g_{1T}^{(n=1)}(k_\perp^2)$$

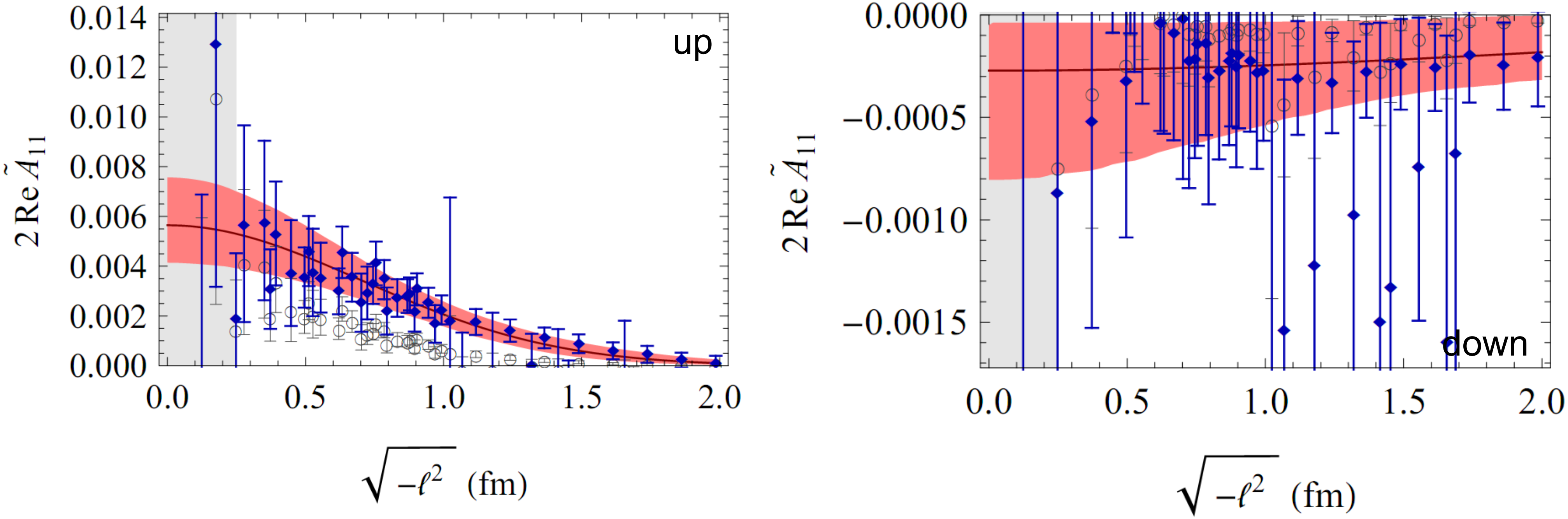
$$\tilde{A}_{10}(l \cdot P = 0, l^2) \leftrightarrow h_{1L}^{\perp(n=1)}(k_\perp^2)$$

$$g_{1T}^{up} \approx -h_{1L}^{\perp,up} > 0$$
$$g_{1T}^{down} \approx -h_{1L}^{\perp,down} < 0$$

comp. to LC quark model:
Pasquini et al PRD 2008

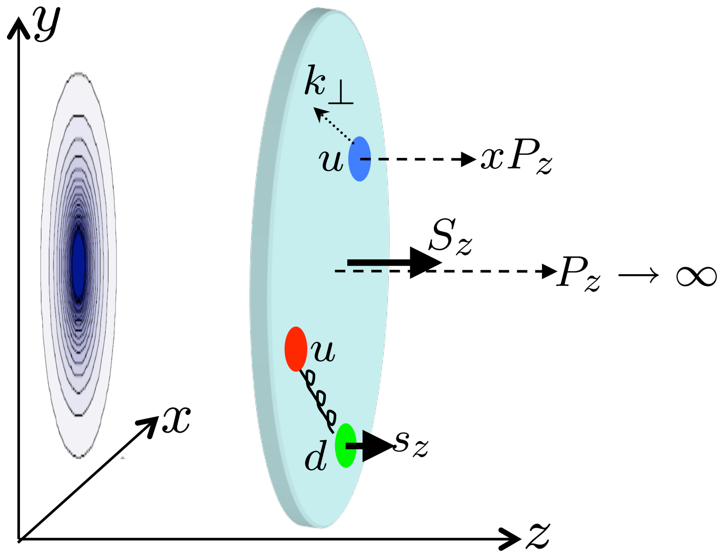
Invariant amplitudes related to quadrupole deformations

(„pretzelosity")



$\leftrightarrow h_{1T}^{\perp,up} < 0,\ h_{1T}^{\perp,down} > \approx 0$

Intrinsic transverse momentum densities of the nucleon



$$\rho_L(x,\boldsymbol{k}_\perp;\Lambda,\boldsymbol{S}_\perp,\lambda) = \frac{1}{2}\left(f_1 + \lambda\Lambda g_1 + \left[\frac{\boldsymbol{S}_j\epsilon_{ji}\boldsymbol{k}_i}{m_N} f_{1T}^\perp\right] + \lambda\frac{\boldsymbol{k}_\perp\cdot\boldsymbol{S}_\perp}{m_N} g_{1T}\right)$$

Diehl, PhH EPJC 44 (2005)

$$\rho_T(x,\boldsymbol{k}_\perp;\Lambda,\boldsymbol{S}_\perp,\boldsymbol{s}_\perp) = \frac{1}{2}\left(f_1 + \boldsymbol{s}_\perp\cdot\boldsymbol{S}_\perp h_1 + \left[\frac{\boldsymbol{s}_j\epsilon_{ji}\boldsymbol{k}_i}{m_N} h_1^\perp\right]\right.$$
$$\left. + \Lambda\frac{\boldsymbol{k}_\perp\cdot\boldsymbol{s}_\perp}{m_N} h_{1L}^\perp + \frac{\boldsymbol{s}_j(2\boldsymbol{k}_j\boldsymbol{k}_i - \boldsymbol{k}_\perp^2\delta_{ji})\boldsymbol{S}_i}{2m_N^2} h_{1T}^\perp\right)$$

Boglione, Mulders PRD 60 (1999)

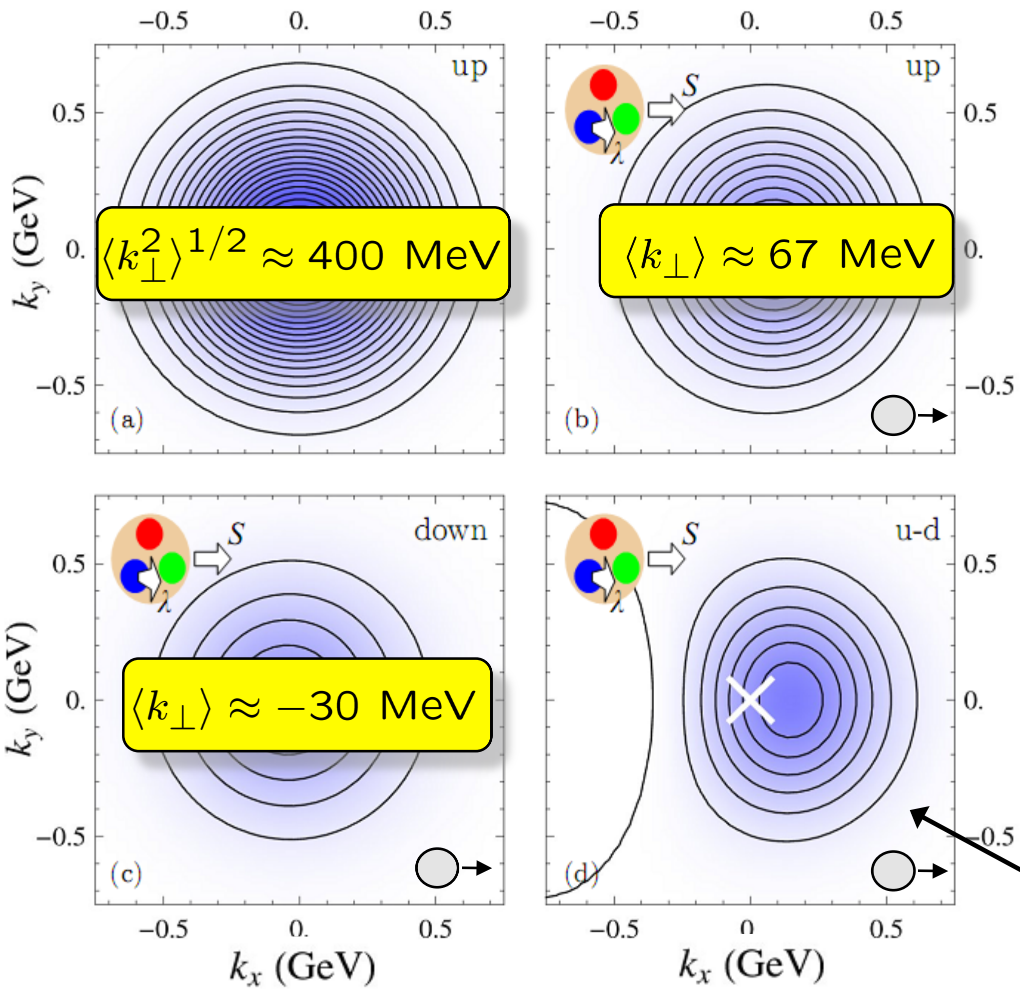
Correlations in

$b_\perp,\ s_\perp,\ S_\perp$

$k_\perp,\ \lambda,\ S_\perp$

Intrinsic transverse momentum densities of the nucleon

$$\rho(x, k_\perp; \lambda, S_\perp) = \frac{1}{2}\left(f_1(x, k_\perp^2) + \lambda \frac{k_\perp \cdot S_\perp}{m_N} g_{1T}(x, k_\perp^2)\right)$$



PhH, B. Musch et al.
arXiv:0908.1283

~~$g_{1T}(x, k_\perp) \leftrightarrow$ GPDs~~

genuine effect
of intrinsic transverse
momentum of quarks

Approximate relations between GPDs and TMDs

T-odd Sivers

$$f_1 \;\leftrightarrow H, \qquad f_{1T}^{\perp} \leftrightarrow \; - E', \qquad g_1 \;\leftrightarrow \tilde{H},$$

$$h_1 \;\leftrightarrow H_T - \Delta_b \tilde{H}_T/(4m^2)\,,$$

$$h_1^{\perp} \leftrightarrow \; - (E'_T + 2\tilde{H}'_T)\,, \qquad h_{1T}^{\perp} \leftrightarrow 2\tilde{H}''_T\,.$$

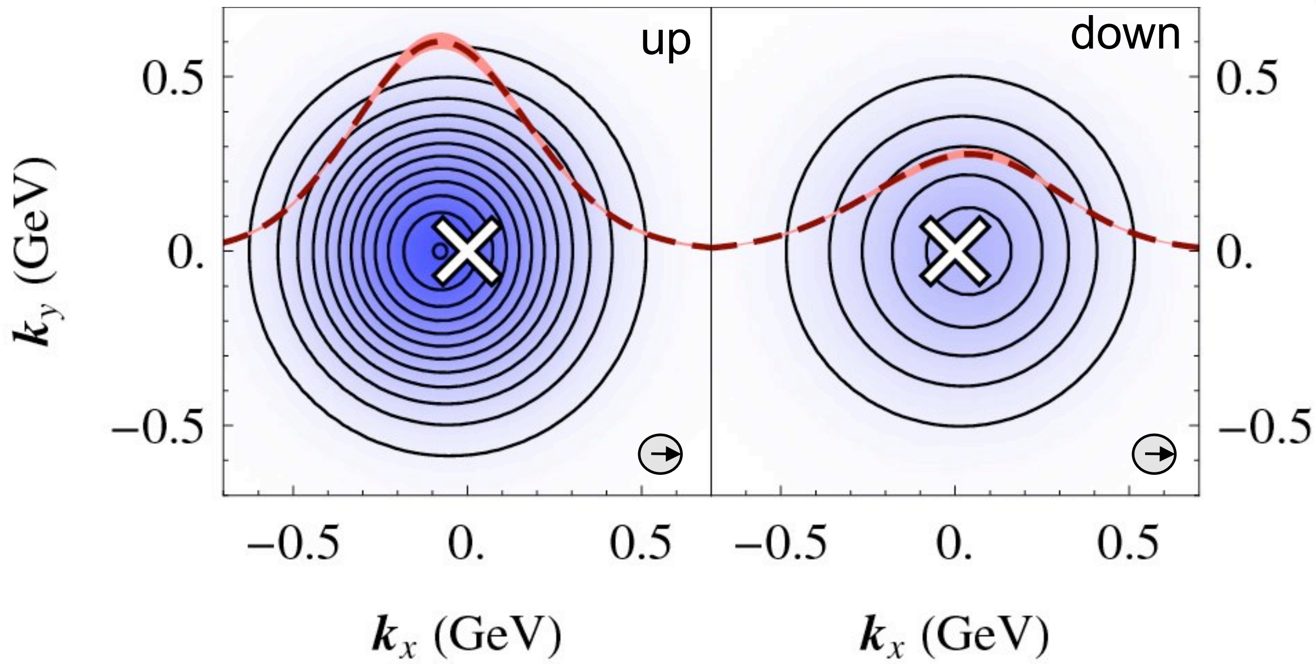
T-odd Boer-Mulders

Burkardt PRD 2002
Diehl, PhH EPJC 2005
Metz et al. 2007

Intrinsic transverse momentum densities of the nucleon

$$\rho(x, k_\perp; \Lambda, s_\perp) = \frac{1}{2}(f_1 + \Lambda \frac{k_\perp \cdot s_\perp}{m_N} h_{1L}^\perp)$$

PhH, B. Musch et al.
arXiv:0908.1283



~~$h_{1L}^\perp(x, k_\perp) \leftrightarrow$ GPDs~~

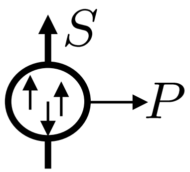
genuine effect of intrinsic transverse momentum of quarks

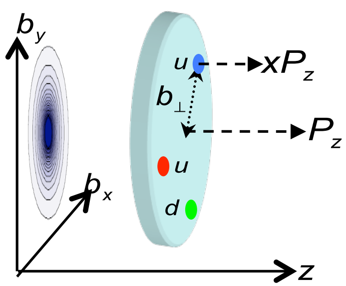
Transverse spin densities in the proton

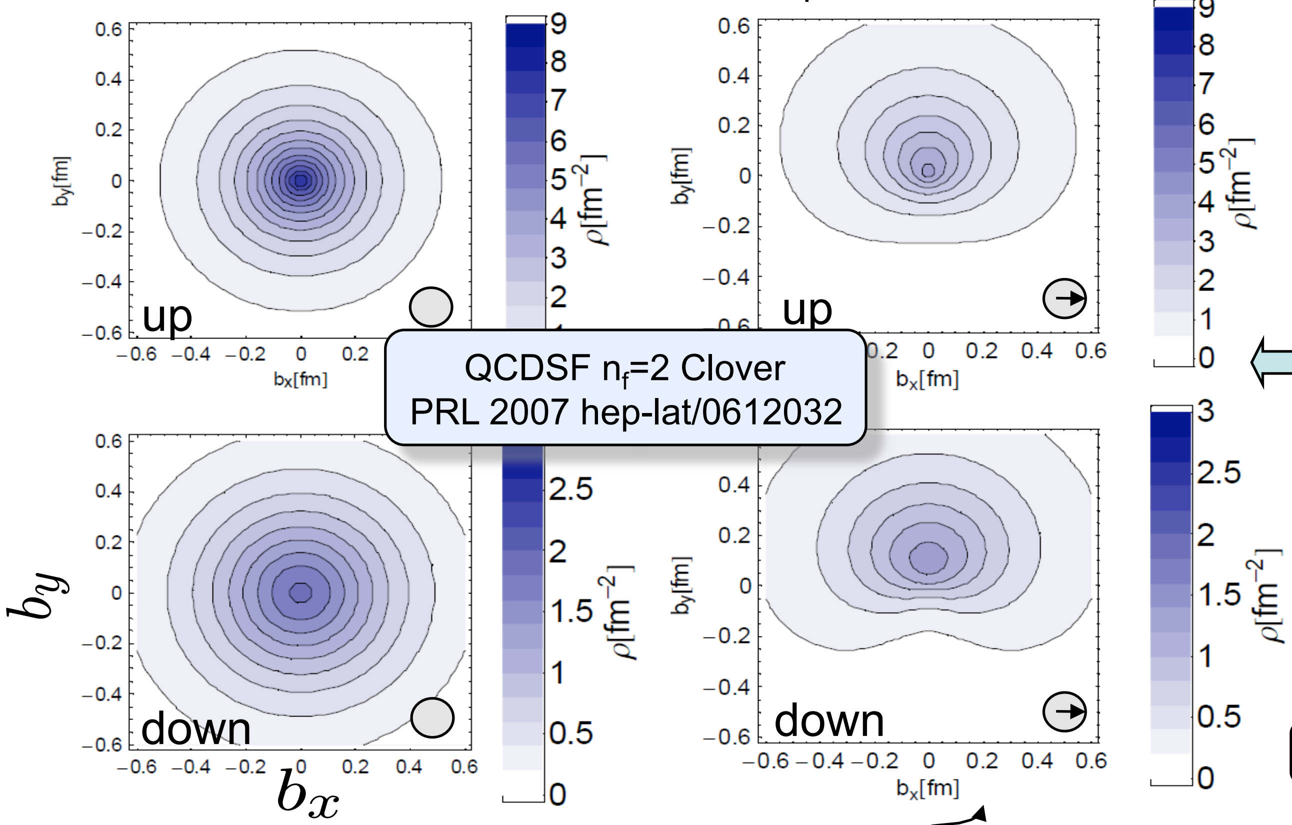
in impact parameter space from GPDs

Diehl / PhH EPJC 2005

$$\langle P^+,0_\perp,S_\perp|\hat{\rho}_T(x,b_\perp;s_\perp)|P^+,0_\perp,S_\perp\rangle = \frac{1}{2}\Big\{H + s_\perp^i S_\perp^i\Big(H_T - \frac{1}{4m^2}\Delta_b\widetilde{H}_T\Big) - \epsilon_{ij}S_\perp^i b_\perp^i \frac{1}{m}E' - \epsilon_{ij}s_\perp^i b_\perp^i \frac{1}{m}\overline{E}'_T + s_\perp^i(2b_\perp^i b_\perp^j - b_\perp^2\delta^{ij})S_\perp^j \frac{1}{m^2}\widetilde{H}''_T\Big\}$$







QCDSF n_f=2 Clover
PRL 2007 hep-lat/0612032

lattice calculations of quark *spin-flip couplings*

$\overline{\text{MS}}$ at 4 GeV^2

strongly deformed transverse spin densities

Correlations between momenta, positions, spins

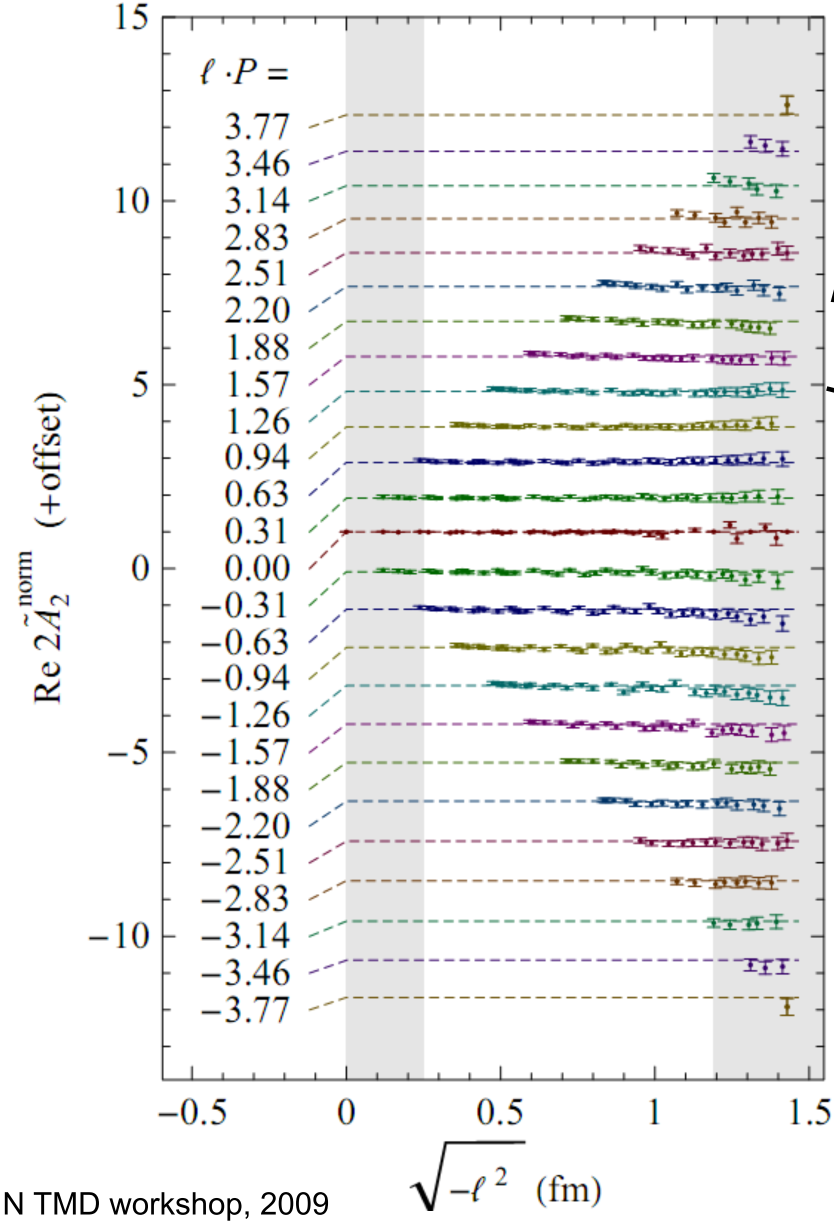
Correlations in

$x,\ b_\perp$ $\qquad$ $x,\ k_\perp$

Transverse momentum dependent PDFs

correlations in x and $k_\perp$

Musch et al. n_f=2+1 mixed tbp and PoS LC2008



$$A_2^{\mathrm{norm}}(l \cdot P, l^2) \equiv \frac{A_2(l \cdot P, l^2)}{A_2(l \cdot P = 0, l^2)}$$

no visible correlations in $l \cdot P$ and l^2

$$k_\perp \leftrightarrow l_\perp$$

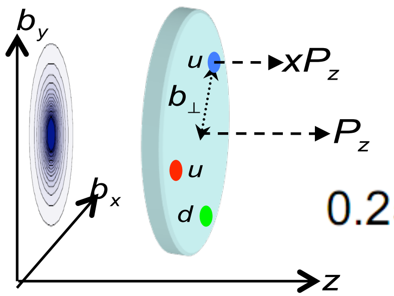
$$x \leftrightarrow l \cdot P$$

≈ factorization of tmdPDFs in x and $k_\perp$

$$f(x, k_\perp) \approx f(x) g(k_\perp)$$

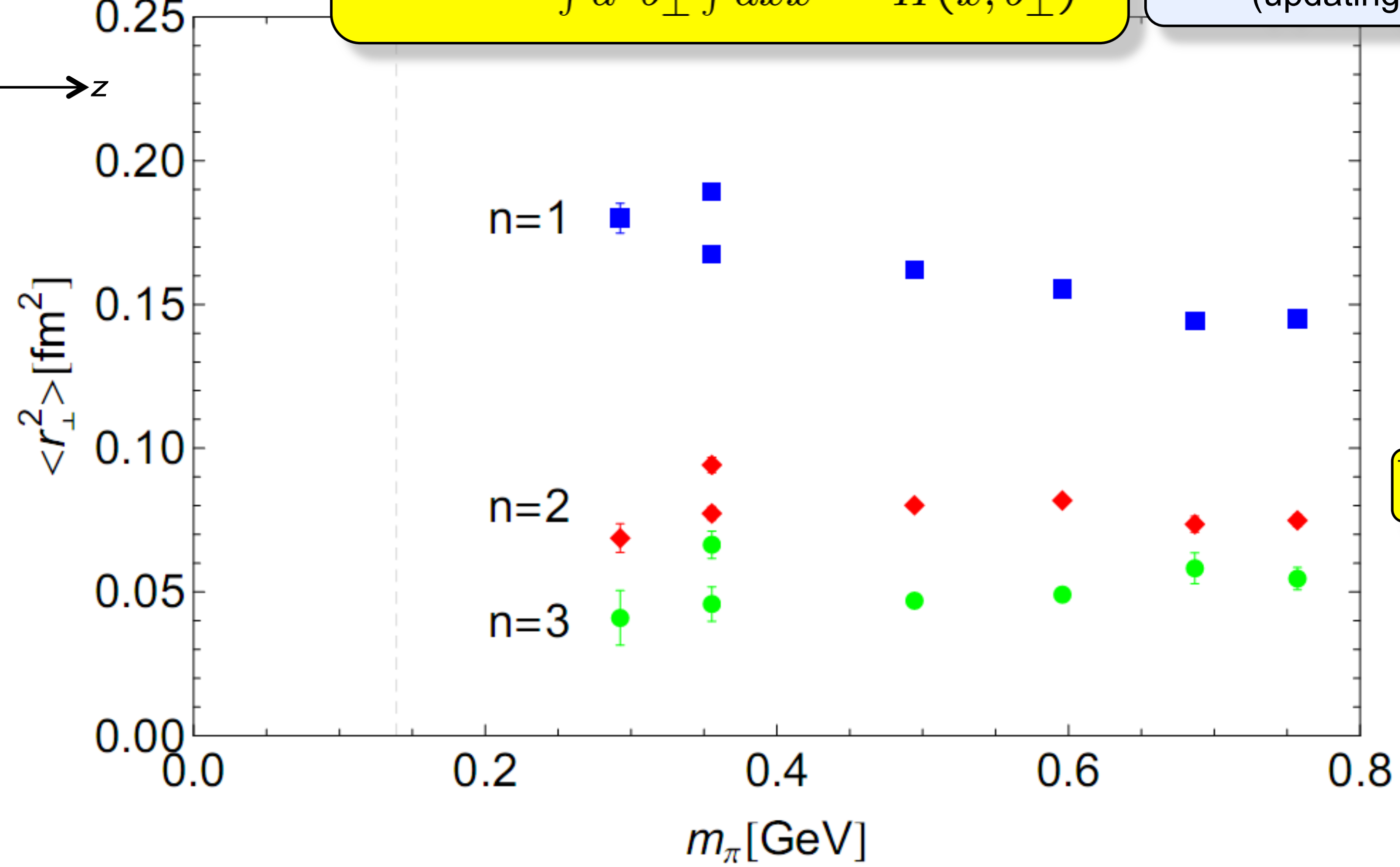
Reminder: Generalized mean square radii of the nucleon

correlations in x and $b_\perp$



$$\langle r_\perp^2 \rangle_n = \frac{\int d^2b_\perp b_\perp^2 \int dx x^{n-1} H(x, b_\perp)}{\int d^2b_\perp \int dx x^{n-1} H(x, b_\perp)}$$

LHPC n_f=2+1 mixed preliminary (updating PRD 2008)



$\overline{\langle x \rangle} \approx 0.2$

$\overline{\langle x \rangle} \approx 0.25$

$\overline{\langle x \rangle} \approx 0.4$

strong correlations in x and $b_\perp$ ⟷ no factorization of GPDs in x and t ⟷ ~~$H(x, 0, t) = f(x)h(t)$~~

Summary

- first direct lattice calculation of non-local operators related to TMDs
  - exploratory calculations based on „process-independent“ direct Wilson lines
    - renormalization of potential power divergencies using string potential
      - Gaussian parametrization of l^2-dependence
        - $k_\perp$-densities of polarized quarks in a polarized nucleon are clearly deformed
          - type of deformation not present for GPDs
      - approximate factorization of $l \cdot P$ and l^2 dependencies
        - in strong contrast to GPDs

analyze additional ensembles — study different link structures — „T-odd“ effects

as always, I am indebted to my collaborators

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References: QCDSF PoS(LAT2006)120, 0710.1534, PRL 98 222001 (2007), PRL 2008 (0708.2249),
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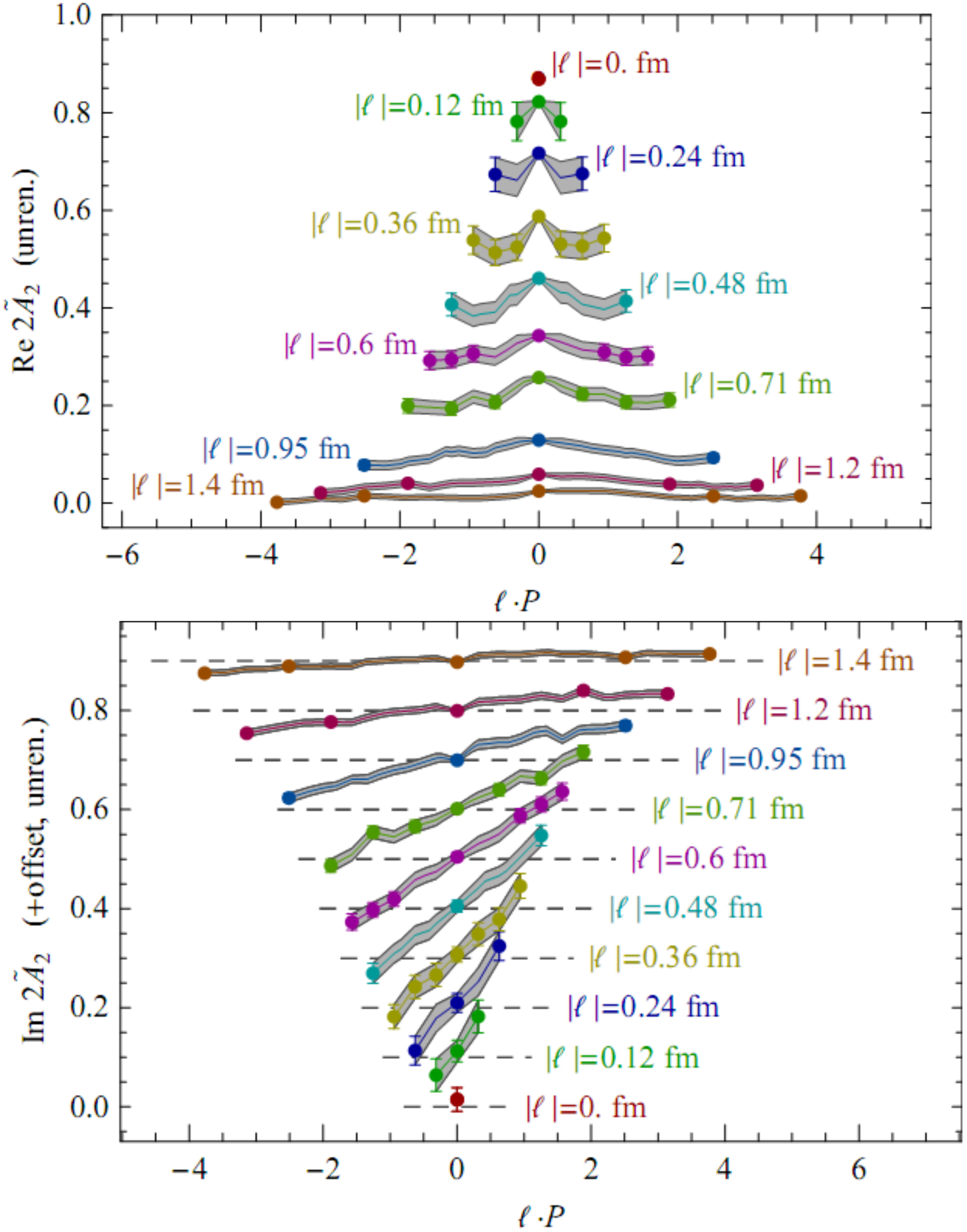
Musch et al. 0811.1536; Musch arXiv:0907.2381; PhH, Musch et al. arXiv:0908.1283



|ℓ |=0. fm
|ℓ |=0.12 fm
|ℓ |=0.24 fm
|ℓ |=0.36 fm
|ℓ |=0.48 fm
|ℓ |=0.6 fm
|ℓ |=0.71 fm
|ℓ |=0.95 fm
|ℓ |=1.2 fm
|ℓ |=1.4 fm
Re $2\tilde{A}_2$ (unren.)
ℓ · P
Im $2\tilde{A}_2$ (+offset, unren.)
ℓ · P